

Facts on F-modules where F is a field , Math 531, Spring 2014

Ayman Badawi

Fact 1. Assume V is an F-module (V is a vector space over F) and $dim(V) = n$. Then

(i) Recall that $v_1, ..., v_k$ are independent if and only if whenever $a_1v_1 + ... + a_kv_k = 0_V$ and $a_1, ..., a_k \in F$, then $a_1 = \cdots = a_k = 0_F$.

(ii) Recall that $v_1, ..., v_k$ are dependent if and only if $a_1v_1 + ... + a_kv_k = 0_V$ for some $a_1, ..., a_k \in F$ such that not all the $a_i's$ are zero.

(iii) Recall we say $\{w_1, ..., w_n\}$ is a basis for V if $w_1, ..., w_n$ are independent and hence every element in V is a linear combination of the $w_i's$.

(iv) Every n independent elements in V form a basis for V over F. i.e., if $v_1, ..., v_n$ are independent, then $span\{v_1, ..., v_n\}$ = V and so every element in V is a linear combination of the $v_i's$.

(v) If D is a subspace of V (i.e., D is an F-module and $D \subseteq V$) and $D \neq V$, then $dim(D) < n$.

(vi) Let D be a subspace of V. Then $D = V$ if and only if dim(D) = dim(V).

(vii) Assume $m > n$, then every m elements in V are dependent.

(viii) If W and D are subspaces of V, then $dim(W) = dim(D)$ if and only if D and W are F-isomorphic as F-modules.

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com